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GENERALIZATION OF THE POSTULATES OF SPECIAL RELATIVITY

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GENERALIZATION OF THE POSTULATES OF SPECIAL RELATIVITY

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ABSTRACT

The suggestion is made that the general particle-wave velocity equation $v_{group}V_{phase} = c^2$ should be taken as a basic postulate of special relativity, rather than the more limited electromagnetic velocity equation $v_{group} = V_{phase} = c$ that is customarily invoked. The equation $vV = c^2$ applies to both massive and massless systems, as does the special theory of relativity itself, whereas the equation $v = V = c$ applies only to massless systems. A relativistically-spinning sphere that exhibits de Broglie's "internal particle frequency" ω_0 is described, and its Lorentz transformation properties are calculated numerically.

Einstein founded the special theory of relativity on the basis of two postulates, which can be stated as follows[1,2]:

<u>Postulate 1.</u> The physical laws of electromagnetism and mechanics are covariant (invariant in form) for transitions from one inertial observer to another inertial observer.

<u>Postulate 2.</u> The speed of light in empty space has the value c as measured in all inertial frames.

However, these two postulates are not on the same footing. Postulate 1 applies

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to both massive particles (which possess rest masses) and massless photons (which do not), whereas Postulate 2 only applies to massless photons. The Lorentz equations of special relativity were originally formulated in order to account for electromagnetic (optical) phenomena. But these equations, which follow directly from Postulates 1 and 2, also apply to massive particles, where they give rise (for example) to the relativistic mass increase and the relativistic dilation of decay times. With the addition of de Broglie particle waves, the relativistic extension from electromagnetic particle-wave systems to massive particle-wave systems becomes complete. Thus it seems desirable to make the basic postulates of special relativity broad enough to encompass both types of system, which requires a generalization of Postulate 2.

A precautionary comment should be made here. Postulates 1 and 2 are concerned with special relativity and with electromagnetic phenomena (where special relativity is known to apply). However, de Broglie particle waves are regarded as a <u>quantum</u> phenomenon. Thus our proposed generalization of the relativity postulates appears to involve an extension of these postulates into the domain of quantum mechanics, where the application of special relativity is much less certain. As a justification for this generalization, we note the well-known fact, which we outline below, that the particle-wave velocity relationship $vV = c^2$ is in fact <u>mandated</u> in order to insure the Lorentz invariance of a particle and its accompanying plane wave <u>irrespective of the nature of the wave</u>. This suggests that the velocity relationship $vV = c^2$ may in fact be a purely special relativistic effect, and there are calculational results which support this viewpoint[3].

In line with the above comment, we now review the way in which the Lorentz equations enter into the invariance of massive particle-wave systems. The counterpart of a linearly-moving photon and its accompanying electromagnetic wave is a linearly-moving massive particle and its accompanying de Broglie plane wave. The photon-wave system leads, via Postulates 1 and 2, directly to the Lorentz equations. If we similarly require a massive particle-wave system to obey Postulate 1, then, with a suitable extension of Postulate 2, we are also led to the Lorentz equations. We can demonstrate this result by applying the Lorentz equations to the particle-wave system, as is done for example in Møller's book[4], and then examining the consequences. Consider a massive particle p and its associated plane wave ψ that move with colinear velocities v and V, respectively, in an inertial frame S. We assume for simplicity that the velocity vectors $\vec{v}$ and $\vec{V}$ lie in the x,y plane, with each vector inclined at an angle θ to the x axis. The plane wave can be written in the form

$$\psi = A \cos 2\pi F, \qquad F = \nu\{t - (x \cos\theta + y \sin\theta)/V\}, \tag{1}$$

where ν is the frequency of the wave. The phase F is an invariant in all inertial frames[4]. We next consider p and ψ as viewed in a frame of reference S' that is moving with velocity u in the x direction with respect to S. By applying the Lorentz transformation law of velocities to the particle velocity $\vec{v}$ and the Lorentz coordinate transformations to the wave phase F, we can establish relationships between the unprimed (S) and primed (S') quantities for p and ψ. This procedure[4] gives the following equations, first for p:

$$\tan\theta' = \sin\theta/\gamma(\cos\theta - u/v), \tag{2}$$

$$v' = v\{1 - 2u \cos\theta/v + u^2/v^2 - u^2\sin^2\theta/c^2\}^{\frac{1}{2}}/(1 - uv \cos\theta/c^2); \tag{3}$$

and then for ψ:

$$\tan\theta' = \sin\theta/\gamma(\cos\theta - uV/c^2), \tag{4}$$

$$V' = (V - u \cos\theta)/\{1 - 2uV \cos\theta/c^2 + u^2V^2/c^4 - u^2\sin^2\theta/c^2\}^{\frac{1}{2}}, \tag{5}$$

$$\nu' = \gamma\nu(1 - u \cos\theta/V), \tag{6}$$

where $\gamma = (1 - u^2/c^2)^{-\frac{1}{2}}$. If we now set $v = c^2/V$, the particle transformations of eqs. (2) and (3) become identical with the wave transformations of eqs. (4) and (5). Thus a particle-wave system is Lorentz invariant if and only if the particle and wave have the velocity relationship

$$vV = c^2 . \tag{7}$$

Hence if eq. (7) is valid for particle-wave systems, then the Lorentz equations are mandated in order to satisfy Postulate 1. But eq. (7) is in fact a general result that emerges directly from the Planck quantization equation and the de Broglie wavelength equation. We can write these two equations as follows:

$$E \text{ (particle)} = \hbar\omega \text{ (wave)}, \tag{8}$$

$$p \text{ (particle)} = \hbar/\lambda\!\!\!^{-} \text{ (wave)}, \tag{9}$$

where (E,ω^{-1}) and $(p,\lambda\!\!\!^{-})$ are canonical sets of variables. Dividing eq. (8) by eq. (9), we obtain

$$E/p \text{ (particle)} = c^2/v = \omega\lambda\!\!\!^{-} \text{ (wave)} = V, \tag{10}$$

so that the velocity relationship $vV = c^2$ follows immediately as a rigorous result for all particle-wave systems (massive or massless). This leads in turn to the result that the Lorentz equations are required for all particle-wave systems (massive or massless) in order to obtain compliance with Postulate 1.

On the basis of these results, we conclude that Postulate 2 should be generalized as follows:

<u>Postulate 2'.</u> The particle-wave velocity product in empty space has the value $vV = c^2$ as measured in all inertial frames.

This generalization of the postulates changes nothing mathematically in the special theory of relativity, but it serves to highlight some of the underlying physical concepts. For example, it indicates the now-well-recognized fact that the particle-wave duality has the same physical reality for both massive and massless systems. Also, it draws attention to de Broglie's "internal particle frequency" ω_o, which is often overlooked in discussions of special relativity, and which we now briefly consider.

The basic relativistic distinction between massive and massless particles is that the former have rest-frame representations whereas the latter do not. In fact, one of the big advantages in dealing with massive particles is that we can mathematically transform from the laboratory frame of reference into the rest frame of the particle. When we do this, the wave frequency ω becomes transformed into the internal particle frequency $\omega_o = \omega/\gamma$, where $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$. This suggests that the frequency ω_o should be accorded physical reality. Furthermore, when we transform back into the laboratory frame of reference, we not only recover the frequency ω of the particle wave, but we also obtain a frequency $\omega_p = \omega_o/\gamma$ for the particle itself that arises from time dilation. The frequency ω_p figured prominently in de Broglie's calculations[5], and he demonstrated that if a linearly-moving particle $p(v,\omega_p)$ and wave $\psi(V,\omega)$ are initially in phase, they remain in phase. This indicates that the particle is in fact the generator of the wave. Extending these ideas to include circular orbits, de Broglie obtained the Bohr orbital quantization rule.

The above discussion leads to the following set of relativistic equations for a particle $p(v,\omega_p,\lambda_p)$ and its associated wave $\psi(V,\omega,\bar{\lambda})$:

$$\begin{aligned} v &= c^2/V \\ \omega_p &= \omega_o/\gamma = \omega/\gamma^2 \\ \lambda_p &= (\gamma^2 - 1)\bar{\lambda} \end{aligned} \qquad \begin{aligned} \omega_o &= m_o c^2/\hbar \\ \gamma &= (1 - v^2/c^2)^{-\frac{1}{2}} \end{aligned} \tag{11}$$

Since these equations are in essence anchored on the rest-frame frequency ω_o together with the requirement of relativistic invariance for p and ψ, a crucial task for relativists is to identify the frequency ω_o with known rest-frame properties of a particle. We now describe a prototype particle model, the relativistically-spinning sphere, that properly exhibits the frequency ω_o.

We start with a spinning sphere in a (non-rotating) rest frame S_o, and we assign it the Compton radius

$$R = \hbar/m_o c \ . \tag{12}$$

We further assume that the sphere is spinning at the frequency ω_o shown in Eq. (11), so that its equator is moving at (or infinitesimally below) the limiting velocity c. Thus the total energy of this spinning sphere is $E_o = m_o c^2 = \hbar c/R = \hbar\omega_o$. The spinning motion produces changes in the initially uniform mass distribution of the sphere. These mass changes can be attributed either to the instantaneous velocities v_m of the mass elements of the sphere as calculated in special relativity, or, equivalently, to the effective gravitational potential that operates in a rotating system[6]. In either case, they cause a rotating circular ring of matter of radius r_m to have an effective spinning mass

$$m_o^r = m_{oo}^r/(1 - v_m^2/c^2)^{\frac{1}{2}} = m_{oo}^r/(1 - \omega_o^2 r_m^2/c^2)^{\frac{1}{2}} \ , \tag{13}$$

where m_o^r is the mass of the ring as observed in S_o, and m_{oo}^r is the rest mass. Integration of these mass elements over the volume of the sphere gives the spinning mass $m_o = 3/2\ m_{oo}$. It also gives the spinning moment of inertia $I = 3/4\ m_{oo}R^2 = \frac{1}{2}\ m_o R^2$. This is a key result, because it leads to the calculated spin angular momentum $J = I\omega_o = \frac{1}{2}\ m_o R^2 \omega_o = \frac{1}{2}\ \hbar$. If a unit electric charge e is placed on the spinning sphere and allowed to move freely, it will be magnetically forced to the equator, where it gives rise to a magnetic moment (in c.g.s. units) $\mu_o = \pi R^2 \cdot i = \pi R^2 \cdot e/c \cdot \omega_o/2\pi = e\hbar/2m_o c$. Thus we see that the rest-frame frequency ω_o in Eq. (11), which was merely postulated by de Broglie[5], can in fact be directly related to both the spin and magnetic moment of the particle via this spatially-extended spinning sphere model.[7]

One test that can be applied to this relativistically-spinning sphere is to see if the mass m_o, spin J, and magnetic moment μ_o tranform properly from the rest frame S_o into the laboratory frame S_ℓ. In Newtonian physics, these three quantities would be invariant. However, experimentally we require [8]

$$(m_o, J, \mu_o) \text{ in } S_o \rightarrow (\gamma m_o, J, \mu_o/\gamma) \text{ in } S_\ell \ , \qquad (14)$$

where the relativistic parameter γ reflects the particle velocity in S_ℓ. As observed relativistically in S_ℓ, the spinning sphere is quite distorted, with a non-uniform azimuthal mass distribution (except for spin orientations along the line of motion), and with a non-spherical shape. Thus its overall transformation properties cannot be obtained analytically. However, they can be determined numerically. In order to accomplish this, we carried out the following calculation. The sphere was represented as a collection of approximately 20,000 discrete spatial elements[9]. Each element was assigned mass, coordinate, and velocity values in S_o. The equatorial spatial elements were also assigned fractional electric charges whose total added up to the unit charge e, so that they formed a representation of the equatorial current loop described above. The element mass values were fixed by matching the spinning mass m_o of the sphere to its Compton radius (Eq. 12). The spin angular momentum J and magnetic moment μ_o in S_o were then numerically calculated, giving the absolute values $J \approx \frac{1}{2}\hbar$ and $\mu_o \approx e\hbar/2m_o c$, in agreement with the above discussion. Next, a Lorentz transformation was individually applied to each of the spatial elements, and its contribution to the mass, spin, and magnetic moment were recalculated in the laboratory frame S_ℓ as functions of the particle velocity $\beta = v/c$ and the spin orientation angle θ (in the rest frame S_o) relative to the line of motion. By summing over the contributions of these 20,000 spatial sphere elements, we obtain relativistically-accurate lab-frame values for the mass, spin, and magnetic moment of this spatially-extended spinning and moving sphere.

In order to more fully understand the workings of the Lorentz transformation process for extended spinning objects, we added one additional feature to these numerical calculations. The Lorentz transformation involves both a coordinate transformation (the relativistic contraction of length along the line of motion) and a velocity transformation (the relativistic velocity addition law). To separate these two relativistic effects, we carried out calculations for three different cases: (1) a relativistic coordinate transformation, with a non-relativistic (Newtonian) velocity transformation (with mass values held constant); (2) a non-relativistic coordinate transformation (no foreshortening), with a relativistic velocity (and mass) transformation; (3) a fully relativistic Lorentz transformation. Cases (1) and (2) are of no direct physical significance, but they delineate the manner in which the Lorentz equations operate for a spatially-extended particle. The results of these calculations are summarized in Table I.

As can be seen in Table I, the mass of the spinning sphere is correctly transformed. However, the spin angular momentum and magnetic moment are correctly transformed only for a spin angle of 0°, and are transformed with approximate accuracy at non-zero spin angles. At these non-zero angles, the relativistic coordinate and velocity contributions to the spin transformation are of opposite signs and are almost equal to one another, so that the resultant spin angular momentum is approximately constant, as required experimentally. The relativistic coordinate and velocity contributions to the magnetic moment transformation, on the other hand, are of the same sign, and they combine together to give approximately the correct $1/\gamma$ dependence for the magnetic moment. The mass transformation of course depends only on the relativistic velocity transformation. In spite of the approximate nature of these results, it seems plausible to conclude that the Lorentz transformations uniquely reproduce the relativistic transformation properties of a spatially-extended massive spinning particle, as embodied in Eq. (14).

The Lorentz equations were originally devised in order to account for the invariance properties of electromagnetic phenomena, as summarized in Postulates 1 and 2. However, these same equations are required in order to reproduce the invariance properties of a massive free particle and its associated plane wave[4], as summarized in Postulates 1 and 2'. Furthermore, these Lorentz equations are needed in order to obtain the proper spectroscopic transformation properties of a spinning particle, as given in Eq. (14). Thus the Lorentz equations are intimately related not just with electromagnetic phenomena, but also with all aspects of massive particles and their associated waves.[3] This suggests that relativity theory should be formulated so as to naturally include both massless and massive particles, which is an argument in favor of Postulates 1 and 2' rather than sub-Postulates 1 and 2 as the basic tenets of special relativity.

In order to give a physical meaning to the de Broglie internal particle frequency ω_o, which serves as the cornerstone for the relativistic relationships summarized in Eqs. (11), we invoked a specific model: the spatially-extended relativistically-spinning sphere. This spinning sphere reproduces the standard rest-frame quantities $R = \hbar/m_o c$, $E_o = \hbar\omega_o = m_o c^2$, $J = \frac{1}{2}\hbar$, and $\mu_o = e\hbar/2m_o c$, and it properly transforms m_o, J, and μ_o (all of which depend on ω_o) into the laboratory frame of reference. Thus it is apparent that a spatially-extended particle is consistent with the dictates of special relativity. It also seems clear that the constraints listed here are sufficient to essentially uniquely determine the parameters of the model. Planck's constant $\hbar$, which is the characteristic feature of the equations of quantum mechanics, enters here via the Compton radius $R = \hbar/m_o c$ as the empirical ratio between the mass and radius of an elementary (non-composite) spin $\frac{1}{2}$ particle.

Table I. Calculated transformation properties of the relativistically-spinning sphere. The relativistic mass m_o, spin angular momentum $J = \frac{1}{2}\hbar$, and magnetic moment $\mu_o = e\hbar/2m_oc$ are first calculated in the rest frame S_o, using 20,000 discrete spatial elements to represent the sphere. Then a series of relativistic (R) and Newtonian (N) transformations to the lab frame S_ℓ are made for each of these elements, as discussed in the text. The results are quoted here as percentage changes from the rest-frame values. θ is the angle between the spin axis and the direction of motion, and β is the relative particle velocity in S_ℓ. The "exp" values shown here for comparison purposes are from Eq. (14).

$\beta = v/c$	θ	Transformations Coord.	Velocity	Mass	Spin	Magnetic moment
		N	N	0.00%	0.00%	0.00%
0.3	0^o	R	N	0.00%	0.00%	0.00%
		N	R	+4.83%	0.00%	−4.61%
		R	R	+4.83%	0.00%	−4.61%
	30^o	R	N	0.00%	−0.58%	−0.58%
		N	R	+4.83%	+0.60%	−4.14%
		R	R	+4.83%	+0.02%	−4.69%
	60^o	R	N	0.00%	−1.73%	−1.73%
		N	R	+4.83%	+1.80%	−3.14%
		R	R	+4.83%	+0.08%	−4.84%
	90^o	R	N	0.00%	−2.31%	−2.31%
		N	R	+4.83%	+2.40%	−2.65%
		R	R	+4.83%	+0.10%	−4.90%
		"exp"		+4.83%	0.00%	−4.61%
0.7	30^o	R	N	0.00%	−3.57%	−3.57%
		N	R	+40.03%	+4.99%	−26.88%
		R	R	+40.03%	+1.42%	−29.51%
		"exp"		+40.03%	0.00%	−28.59%

References.

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(3) M. H. Mac Gregor: UCRL 91722 (Rev. 1), Jan. 31, 1985.

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(5) M. Jammer: The Conceptual Development of Quantum Mechanics (McGraw-Hill, New York, 1966), p. 243-246.

(6) Ref. 4, p. 318-319.

(7) See M. H. Mac Gregor: Phys. Rev. D9, 1259 (1974), App. B, for a discussion of the relativistically-spinning sphere.

(8) B. Cagnac and J. C. Pebay-Peyroula: Modern Atomic Physics (Wiley, New York, 1975), p. 296, have a comment about the variation of the particle magnetic moment with velocity.

(9) This is the largest number of elements that can be accommodated by the memory of a Livermore CRAY-1 computer.